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Haptic perception of spatial relations

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Abstract. There are some indications that haptic space like visual space is not Euclidean (eg Blumenfeld, 1937 *Acta Psychologica* **2** 125 – 174). In a series of experiments, we investigated the haptic perception of spatial relations in a systematic way. We restricted ourselves to a horizontal plane at waist height. Blindfolded subjects were asked to perform three tasks with their right hand: (i) a reference bar was presented under four different orientations and subjects were asked to rotate a test bar such that it felt to be parallel to the reference bar; (ii) subjects had to rotate two test bars in such a way that they felt collinear; (iii) subjects had to point a test bar in the direction of a marker. Bars and marker could appear at nine different locations. In all experiments large systematic deviations (up to 40°) were made. The deviations strongly correlated with horizontal (right – left) but not with vertical (forward – backward) distance. Subjects showed qualitatively identical trends but the size of the deviations was strongly subject-dependent. In addition, a significant haptic oblique effect was found. These results provide strong evidence that haptic space in non-Euclidean.

1 Introduction

In 1937 Blumenfeld published a paper about the haptic and the optical structure of space. The part of most relevance here is a description of a haptic experiment in which he measured the so-called 'alley curves'. Subjects were required to haptically produce parallel lines on both sides of the median plane. Interestingly, the results were far from veridical: the lines produced diverged towards the subject as long as the distance between the lines was less than the distance between the shoulder joints; above this distance the lines gradually became parallel and, for some subjects, even converging. Notwithstanding the fact that Blumenfeld discussed his results in terms of 'the parallelity laws', he did not give a formal description of his results.

In the visual domain, Blumenfeld's experiments (and those of Hillebrand) gave rise to a number of papers, both theoretical (eg Luneburg 1950; Yamazaki 1987) and experimental (eg Battro et al 1976; Indow and Watanabe 1984). Although the results of Blumenfeld's haptic experiments were just as convincingly nonveridical as those of his optical experiments, they never really got a follow-up; only two papers seem to be of any relevance. In 1951 Worchel published a paper about haptic space perception, but in the context of this paper 'space' was more related to tactual form and to spatial relations between forms than to 'space' as meant by Blumenfeld. Probably inspired by the theories of Luneburg (1950), Brambring (1976) investigated the metric of haptic space from the point of view of the geographical orientation of the blind. By asking for distance estimates under various experimental conditions, he established that subjects show significant deviations from the Euclidean metric. Given his experimental paradigms and his aims, he could not, of course, say anything about the way haptic space is distorted. Thus, till the present day, our knowledge of the structure of haptic space remains scarce.

One of the major aims of this paper was to continue experimental work on haptics along the lines initiated by Blumenfeld. Using a number of different experimental paradigms, we investigated whether haptic spatial relations are indeed non-Euclidean by nature. If the visual findings find a parallel in the haptic domain, we would expect the deformations of haptic space to be far from random. We set up our experiments in such a way that if the results were indeed nonveridical and deviated in a systematic way, our data would allow us to find a quantitive description of the deformations.

We performed three different kinds of experiments: (i) a task in which subjects had to make two stimuli parallel; (ii) a task in which subjects had to make two stimuli collinear; and (iii) a pointing task. Apart from analysing the results of each individual task, we were also concerned with the question whether the deformations (if any) found in the various tasks could be attributed to the same underlying mechanism.

Although our aim was similar to that of Blumenfeld (1937), it should be noted that our experiments were different from his in almost all aspects. First, he used a bimanual task whereas we have chosen, as a first step, unimanual tasks. Second, his stimuli were presented symmetrically on both sides of the median plane. In our experiments, the stimuli were always on, or on the right side of, the median plane at different distances from the subject. Third, his stimuli could not be 'touched' in the common sense of the word since they consisted of threads which had to be moved in a parallel fashion. Our stimuli, on the other hand, consisted of aluminium bars, and subjects could actually touch them in various ways (statically or dynamically, with one or more fingers or with the whole hand, etc). Fourth, in Blumenfeld's alley experiment, all stimuli had to be made parallel to the median plane, whereas we used a number of different reference orientations. Finally, our pointing and collinearity experiments did not have an analogue in the work of Blumenfeld.

In the literature, the existence of haptic oblique effects has been frequently reported (eg Lechelt et al 1976; Lechelt and Verenka 1980; Appelle and Gravetter 1985; Appelle and Countryman 1986; Gentaz and Hatwell 1995). Although our experiments were not meant as a quest after a possible haptic oblique effect, our data from the parallelity experiment allow an analysis as a function of the reference orientation. If there exist haptic oblique effects, they should manifest themselves in a worse performance with oblique reference orientations (45° and 135°) in comparison with horizontal (0°) and vertical (90°).

2 Experiment 1

In this first experiment—the so-called 'parallelity' experiment—we investigated what subjects haptically perceive as parallel orientations at different locations. Both the veridicality and the accuracy are of interest here. By veridicality we mean how well the average settings of a subject correspond to settings we would get if haptic space were Euclidean. By accuracy, on the other hand, we mean how well a subject reproduces his/her own settings. Since both veridicality and accuracy might depend on the test positions, nine different locations covering a substantial part of the haptic space within reach of the right hand were tested.

2.1 *Methods*

2.1.1 *Apparatus.* The setup consisted of a large table on which an iron plate of the same size was fixed. This plate was covered by a plastic layer on which fifteen protractors were printed (thus could not be felt), as can be seen in figure 1. In the current experiments, only the nine rightmost protractors (indicated by circles in figure 1) were used. The protractors had a diameter of 20 cm. The spacing of their centres was 30 cm horizontally and 20 cm vertically (in this respect horizontal and vertical are defined as parallel to the long and short sides of the table, respectively). These dimensions were motivated by the size of the area which could be reached with the right hand and our preference to have equal numbers of positions in the horizontal and vertical directions. Two aluminium bars, 20 cm long and 1.1 cm in diameter, were used as the test bar and the reference bar. Each bar had a small pin sticking out halfway along its long axis which fitted exactly in the holes in the centres of the protractors. In this way the bars could be rotated around a fixed position. Small magnets were

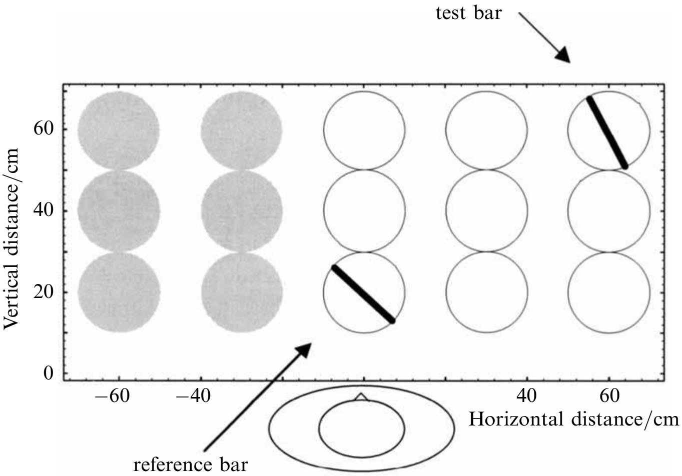

Figure 1. Top view of the setup. The circles indicate protractors printed on the table cover which allow the experimenter to read the orientation of the bars with an accuracy of about 1°. The grey disks are protractors not used in the current experiments. The centres of the two bars can be positioned on the centre of any of the circles. In the pointing experiment, one of the bars is replaced by a marker. The navel of the subjects is always positioned at the coordinates (0, 0).

attached under the bars in order to increase their resistance to movement (hence the use of the iron plate). At both ends of the bars small needles allowed the experimenter an accurate reading (uncertainty of about 0.5°) of the orientation of the bars.

2.1.2 *Stimuli.* For three of the subjects (ME, NK, and RR), all nine positions were used as location for the reference bar. Given a position for the reference bar, the remaining eight locations were used for the test bar. At each location, the reference bar was given orientations of 0°, 45°, 90°, and 135° (0° is parallel to the long side of the table; increasing orientation values signify a rotation in counterclockwise direction). Thus the total number of combinations is 9 (positions of the reference bar) $\times$ 8 (positions of the test bar) $\times$ 4 (the number of reference orientations) $=$ 288. All 288 combinations were presented three times in random order. These orders were different for each of the subjects.

For the three other subjects (RA, MK, and MW) only a subset of the locations was tested. The four corner positions were used as reference locations. Given a reference position, the other three corner positions were used for the test bar. For these subjects, the total number of combinations was 4 (positions of the reference bar) $\times$ 3 (positions of the test bar) $\times$ 4 (number of reference orientations) $=$ 48. All 48 combinations were presented three times in random order.

2.1.3 *Subjects.* Six paid subjects (physics or mathematics undergraduates) participated in the experiment. Subjects ME, NK, MK, and MW were strongly right-handed, subject RR was weakly right-handed, and subject RA was moderately left-handed. Handedness was assessed by means of a standard questionnaire as proposed by Coren (1993). None of the subjects reported haptic deficiencies. Subjects were never shown the setup so they remained unaware of the number of possible test and reference positions and they did not receive any information about the reference orientations. Moreover, the subjects were unfamiliar with the aims of the experiment.

2.1.4 *Procedure.* A blindfolded subject was seated behind the table in such a way that his/her navel was positioned at coordinates (0, 0) (see figure 1). The experimenter positioned the reference bar at the prescribed location and orientation. Next the test bar was positioned at a random orientation. The experimenter then took the right hand of the subject and placed it on the reference bar. Subsequently, the experimenter placed the subject's hand on the test bar. The subject was instructed to rotate the test bar in such a way that it felt as being parallel to the reference bar. The subject was allowed to go back and forth to the reference and test bars as often as needed to be able to perform the task. The subjects were free to choose their own strategy as long as they remained sitting at the correct position. They were, however, not allowed to use their left hand nor to touch the edges of the table. When the subject was satisfied with the orientation of the test bar, the experimenter noted down the orientation. This procedure was repeated for the next trial. The subject was not given any feedback.

Experimental sessions always ended after 1 h to avoid weariness of the subject. Since subjects were allowed to use as much time as they preferred, for each of them a different number of sessions was needed. ME completed this experimental task in 8 h, NK in 17.5 h, and RR in 10.5 h. These experiments were performed over the same period of time as experiments 2 and 3. Subjects RA, MK, and MW who did only a subset of the stimuli needed only 2 h to complete this experiment.

2.2 *Results*

In figure 2 both a graphical and a numerical representation of the results of subject ME are given. In each square, the thick line represents the reference bar and the little dot indicates the position of the subject [coordinates (0, 0)]. The thin lines indicate the orientations of the test bars averaged over the three settings. In all squares, the locations of subject and bars, and the length of bars are drawn to scale. The numbers give the deviation of the average setting of the test bars in degrees. Positive and negative numbers refer to counterclockwise and clockwise deviations, respectively. Along the rows the orientation of the reference bar is constant but its position varies in a systematic way. Along the columns the orientation of the reference bar varies from 0°, via 45° and 90°, to 135°, but its position remains constant.

If subject ME had responded veridically, all thin lines within a square would be parallel to the thick line and all deviations would be zero. Clearly, this is not the case. Deviations as large as 40° occur. Moreover, it can be seen that the deviations form patterns that are far from random. A number of trends can be noticed: (i) going from left to right within a square, the orientations of the lines change mostly clockwise; (ii) the lines within a square are often parallel to each other when compared vertically. With only one exception (RR, trend 2), these observations are true for all subjects. Thus the patterns of deviation of subject ME are representative for the other subjects as well, be it with an important difference: the deviations of the other subjects are smaller.

For subject ME, the average deviation (absolute value) is 11.3°, whereas the average standard deviation (for the three settings) is only 5.2°. This gives a good indication that the orientation of most of the test bars is indeed significantly different from veridical. For the other subjects the average absolute deviations and the average standard deviations over the three settings are the following: NK, 7.2° and 3.4°; RR, 4.8° and 4.7°; RA, 9.3° and 4.3°; MK, 10.5° and 6.2°; MW, 2.3° and 3.3°. One should be careful in comparing the values of the latter three subjects with the other ones since they have been obtained from a subset of stimulus combinations. What is important here is that, except for subject MW who responds almost veridically, the average absolute deviations are larger than the average standard deviations. Only for subject RA does the accuracy (the standard deviation) depend significantly on the position of the test bar ($p < 0.05$).

2.2.1 *Dependence on distance.* In order to analyse the data and the above-mentioned trends in more detail, we plotted the deviations as a function of both horizontal and vertical distances (see figures 3 and 4, respectively). A negative horizontal distance indicates that the test bar is located to the left of the reference bar, and vice versa; a positive vertical distance indicates that the test bar is located further away from the subject in vertical direction than the reference bar, and vice versa. From left to right,

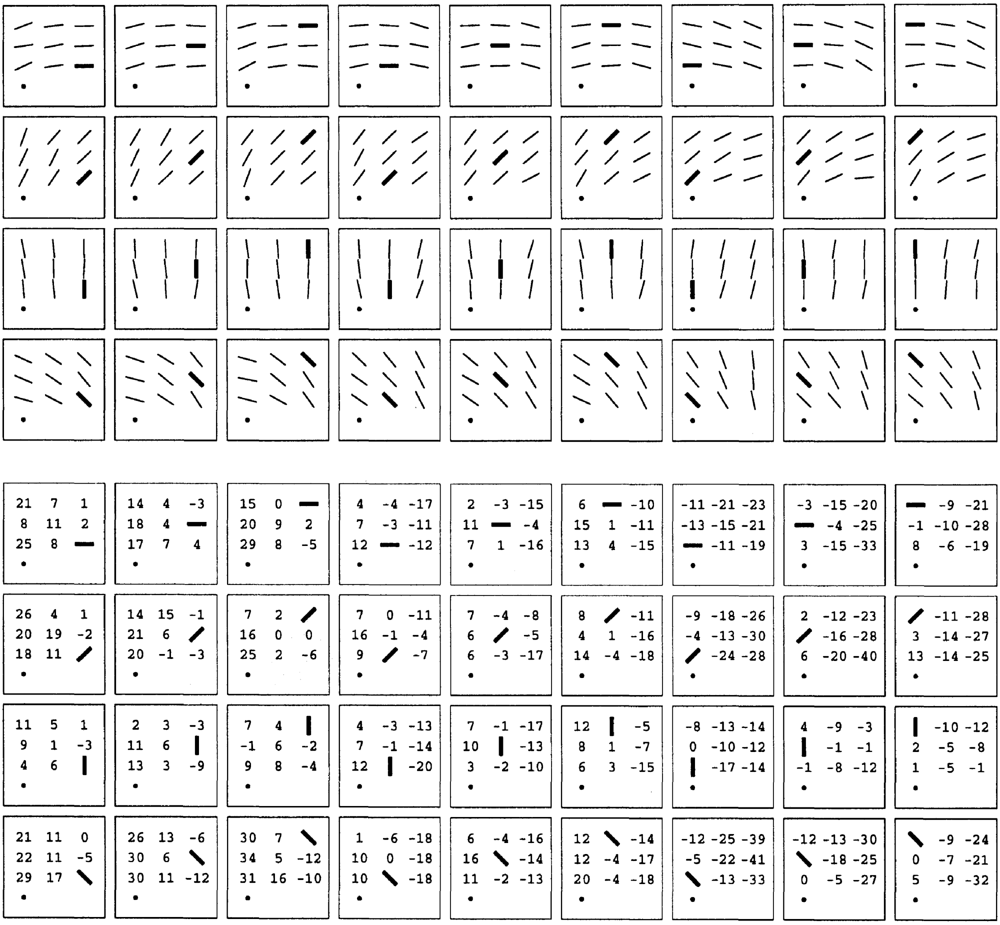

Figure 2. Results of subject ME in the parallelity experiment. The upper four rows give a graphical representation of the results whereas the lower four rows give a numerical representation of the same results. In each of the squares, the small dot indicates the position of the subject and the thick line represents the reference bar. In each row, the orientation of the reference bar is kept constant but the position varies. In rows 1 and 5, the reference orientation is 0°; in rows 2 and 6, 45°; in rows 3 and 7, 90°; in rows 4 and 8, 135°. The thin lines in the upper squares give the orientation of the test bars averaged over the three settings. The numbers in the lower squares give the deviation in degrees of the average settings of the test bars. Positive and negative numbers indicate counterclockwise and clockwise deviations, respectively. If the subject responded veridically, all thin lines within a square would be parallel to the thick line and all deviations would be zero. Locations of subject and bars, and length of bars are drawn to scale.

the columns show the results for reference orientations 0°, 45°, 90°, and 135°, respectively. The rows give the results for the six subjects. The solid line in each graph indicates the line of best fit (in the least-squares sense). The slopes of these lines and the correlation coefficient (r^2) of the data points in a graph are given in table 1. In this table, the columns indicated with "all" give the values for all reference orientations taken together.

The trends observed in figure 2 in the data of subject ME are even more evident in the two figures and the table. For all subjects and almost all reference orientations, there is a significant dependence of the deviation on the horizontal distance between the test bar and the reference bar: the more positive (negative) the horizontal distance, the more negative (positive) the deviation. Within the range of distances measured (–60 cm to +60 cm) this dependence is linear. Conversely, with only a few exceptions there is no

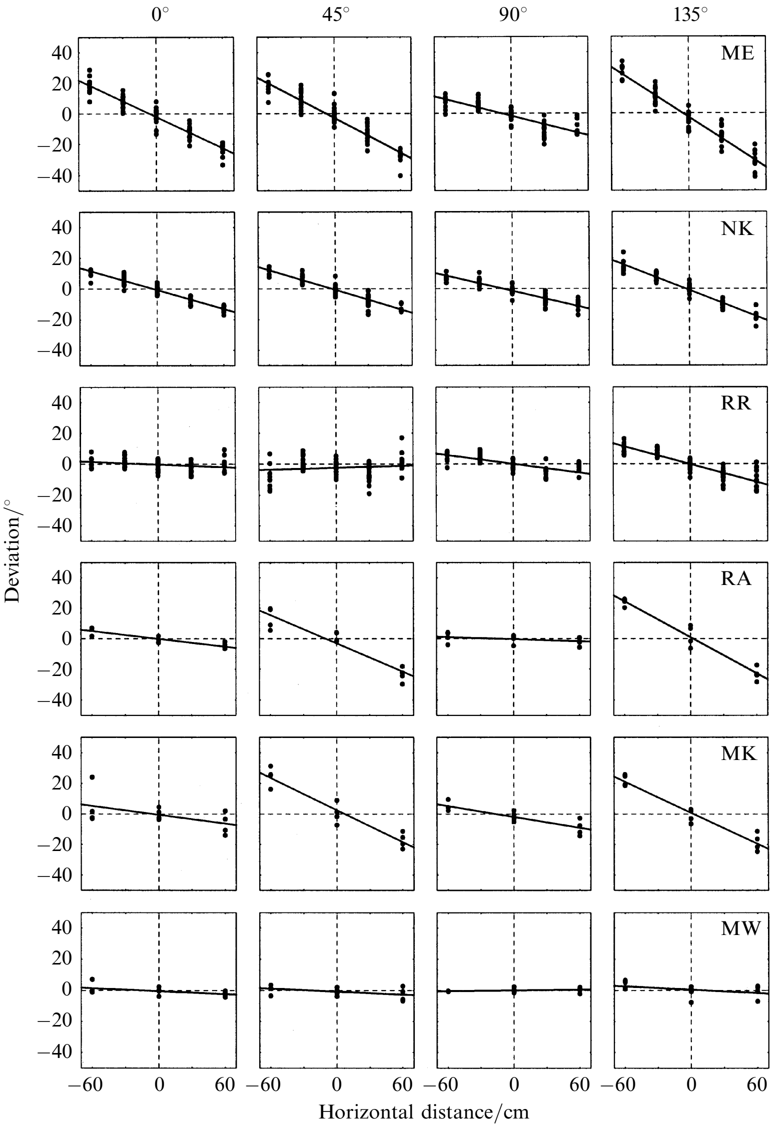

Figure 3. Deviations as a function of the horizontal distance between reference and test bar. A negative distance indicates that the reference bar is located to the right of the test bar, and vice versa. From left to right, the columns give the data for the four reference orientations: 0°, 45°, 90°, and 135°, respectively. The six rows show data of the six subjects. The solid line indicates the best fit (in the least-squares sense) through the data points. Slopes, r^2, and significance levels are given in table 1. As can be seen in this figure and table 1, most data points are highly correlated.

dependence of the deviation on the vertical distance. In this respect it should be noted that the solid lines in figure 4 hardly mean anything since the corresponding correlations (see table 1) are close to zero.

2.2.2 *Dependence on reference orientation.* Both accuracy (standard deviation) and veridicality (deviation from veridical) might depend on the reference orientation. Indeed, in figure 3 and table 1 it can easily be seen that the deviations depend on the reference orientation. Moreover, most strongly in the cases of subjects RA and MK, an oblique effect can be observed: the deviations are larger for oblique reference orientations (45° and 135°) than for the horizontal (0°) and vertical (90°) reference orientations.

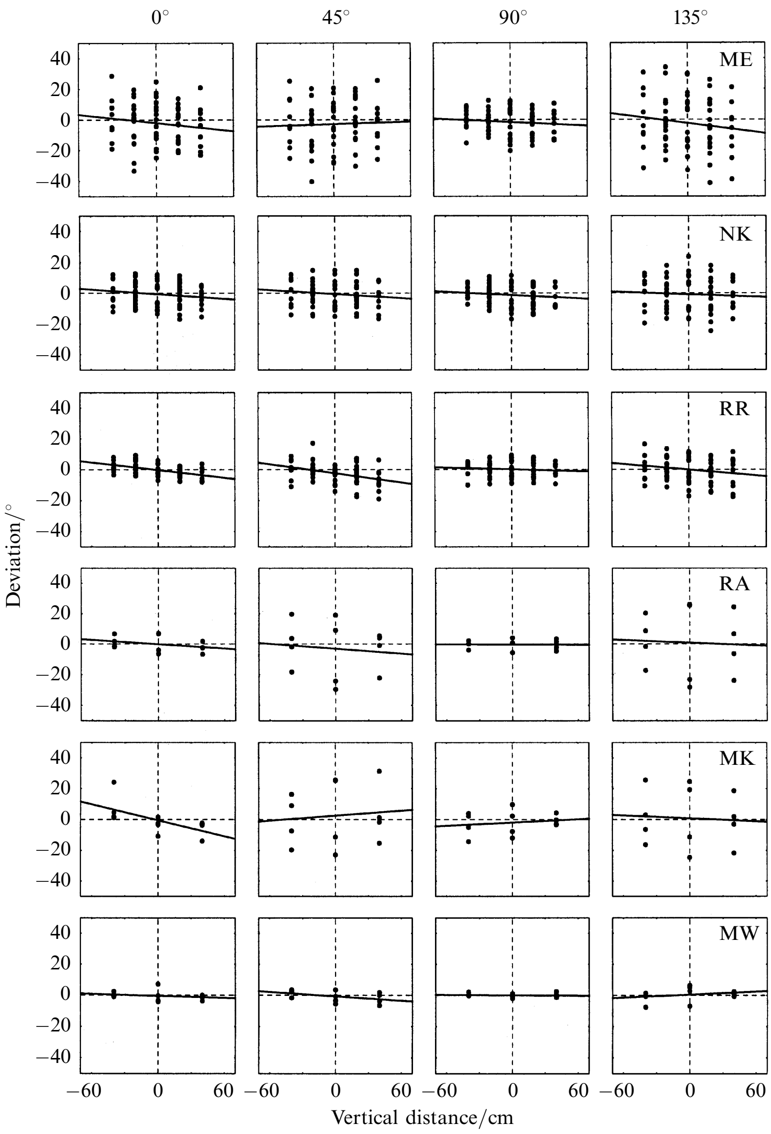

Figure 4. Deviations as a function of the vertical distance between reference and test bar. A negative distance indicates that the reference bar is located further away from the subject in vertical direction than the test bar, and vice versa. From left to right, the columns give the data for the four reference orientations: 0°, 45°, 90°, and 135°, respectively. The six rows show data of the six subjects. The solid line indicates the best fit (in the least-squares sense) through the data points. Slopes, r^2, and significance levels are given in table 1. Both from this figure and table 1, it is clear that the data points are hardly correlated.

In our experiments it does not make sense to look at signed errors for this analysis, since the sign depends on the relative positions of the test and reference bars. Therefore we analysed the absolute errors. We performed a one-way ANOVA on the subjects' data to investigate the significance of the dependence of the absolute error on orientation. For all six subjects, the effect is significant (ME: $F_{3,284} = 12.11$, $p < 0.0001$; NK: $F_{3,284} = 5.86$, $p < 0.001$; RR: $F_{3,284} = 13.44$, $p < 0.0001$; RA: $F_{3,44} = 13.03$, $p < 0.0001$; MK: $F_{3,44} = 5.68$, $p < 0.005$; MW: $F_{3,44} = 2.98$, $p < 0.05$). In order to investigate the oblique effect more directly, we collapsed the data into the two categories horizontal/vertical and oblique. It turns out that for all six subjects the oblique effect is

Table 1. Slopes and r^2 values for the deviations (in degrees) as a function of distance (above: horizontal distance; below: vertical distance) and as shown in figures 3 and 4. 0°, 45°, 90°, and 135° indicate the orientation of the reference bar. In columns "all", the data points obtained for all four reference orientations are taken together. The asterisks indicate the significance level of the slopes: *, $p < 0.05$; **, $p < 0.0001$.

Subject	Slope/° m^{-1}					r^2				
	0°	45°	90°	135°	all	0°	45°	90°	135°	all
(a) Horizontal distance										
ME	−34**	−38**	−18**	−47**	−34**	0.88	0.87	0.66	0.91	0.80
NK	−21**	−21**	−17**	−28**	−22**	0.89	0.87	0.82	0.93	0.86
RR	−3*	2	−9**	−19**	−7**	0.07	0.01	0.49	0.74	0.19
RA	−9**	−31**	−2	−39**	−20**	0.80	0.88	0.14	0.95	0.57
MK	−10	−35**	−12**	−34**	−23**	0.28	0.91	0.74	0.94	0.65
MW	−3	−3	0	−4	−2*	0.28	0.22	0.08	0.19	0.13
(b) Vertical distance										
ME	−8	2	−3	−9	−4	0.02	0.01	0.01	0.02	0.01
NK	−5	−4	−3	−2	−4	0.02	0.02	0.02	0.00	0.01
RR	−8**	−10**	−2	−6	−7**	0.27	0.14	0.01	0.03	0.07
RA	−5	−5	0	−3	−3	0.11	0.01	0.00	0.00	0.01
MK	−17*	5	3	−3	−3	0.39	0.01	0.03	0.00	0.00
MW	−2	−5	0	3	−1	0.06	0.21	0.01	0.07	0.01

significant (ME: $F_{1,286} = 20.79$, $p < 0.0001$; NK: $F_{1,286} = 9.31$, $p < 0.005$; RR: $F_{1,286} = 27.39$, $p < 0.0001$; RA: $F_{1,46} = 35.67$, $p < 0.0001$; MK: $F_{1,46} = 17.69$, $p < 0.0001$; MW: $F_{1,46} = 7.09$, $p < 0.05$). These analyses confirm what could already be seen in figure 3.

Similarly, we performed a one-way ANOVA on the standard deviation to investigate the possible dependence on reference orientation. This analysis reveals a significant effect of orientation for subjects ME, RR, and MW (ME: $F_{3,284} = 4.49$, $p < 0.005$; NK: $F_{3,284} = 1.43$, $p > 0.2$; RR: $F_{3,284} = 4.96$, $p < 0.005$; RA: $F_{3,44} = 1.43$, $p > 0.2$; MK: $F_{3,44} = 0.38$, $p > 0.7$; MW: $F_{3,44} = 6.81$, $p < 0.001$). It should be noticed that for subject ME this effect is due to a higher average standard deviation for orientation 90° instead of a lower one as expected for the oblique effect. If we collapse the data once more into two categories, horizontal/vertical and oblique, we only find a significant effect of orientation for subjects RR and MW (ME: $F_{1,286} = 2.68$, $p > 0.1$; NK: $F_{1,286} = 1.59$, $p > 0.2$; RR: $F_{1,286} = 11.24$, $p < 0.001$; RA: $F_{1,46} = 1.18$, $p > 0.2$; MK: $F_{1,46} = 0.25$, $p > 0.6$; MW: $F_{1,46} = 16.04$, $p < 0.0005$).

3 Experiment 2

In this second experiment—the so-called 'collinearity' experiment—we investigated what orientations two bars at various positions need to have for subjects to haptically perceive them as collinear. Given the results of the parallelity experiment, appreciable deviations were expected. In order to be able to compare the results of the two experiments, the same nine locations were used.

3.1 *Methods*

3.1.1 *Apparatus, stimuli, and subjects.* The same setup as in experiment 1 could be used for the collinearity experiment. Since there were nine positions, there were $(9 \times 8)/2 = 36$ different possible combinations of positions for the two bars (division by 2 is necessary because the two bars are identical). All these 36 combinations were used in this experiment. They were presented three times in random order on different days. Subjects ME, NK, and RR from the first experiment participated here as well.

3.1.2 *Procedure.* The procedure was much the same as that in experiment 1. The experimenter placed the right hand of a blindfolded subject first on one of the bars and then on the other. The subject was instructed to rotate both bars in such a way that it felt as if they were lying on a straight line through both of them. The subject could take as much time as he/she preferred. When the subject indicated that the bars felt collinear, the experimenter noted down the orientations of the two bars. This procedure was repeated for the next trial. The subject was not given any feedback.

The duration of an experimental session was always 1 h. If subjects needed less time for one run of this experimental task, the remaining time of the session was filled with blocks from either experiment 1 or experiment 3. ME needed a total of 1 h to complete this task, NK 4 h, and RR 1.5 h.

3.2 *Results*

In figure 5 the results are shown for subject ME. The 36 upper squares show the results for the 36 different combinations of the test bars in a graphical way; the corresponding lower squares show the same results in a numerical way. In each square the little dot represents the position of the subject [coordinates (0, 0)]. In each of the upper squares, the two lines indicate the settings of the test bars (averaged over the three trials). Locations of subject and bars, and length of bars are drawn to scale. For each given pair of test bars there is only one possible orientation (for both bars) in which they are physically collinear. In the lower squares the deviations in degrees from this physical (veridical) orientation are shown for each bar separately. As in experiment 1, it is clear that the settings deviate in a systematic way from veridical. In most cases, the right bar is rotated clockwise and the left bar counterclockwise with respect to the veridical orientation. This description is true for all three subjects, with the restriction that the sizes of the deviations depend on the subjects. Again, the average of the absolute values of the deviations and the average standard deviations together should give an impression of the significance of the deviations and of differences between the subjects. These values are 5.5° and 3.2° for subject ME, 3.6° and 1.7° for subject NK, and 2.3° and 2.2° for subject RR, respectively. Clearly, the responses of subject RR are again close to veridical, but still the trend in his data is similar to those of subjects ME and NK.

In figure 6 we plotted the deviations as a function of horizontal (upper graphs) and vertical (lower graphs) distances. For the deviations we took the difference in orientation between the two bars [if they are collinear, the deviation would be zero; we always subtracted the orientation of the leftmost bar from the other one (or, for the vertical distances, the orientation of the lower bar from the orientation of the upper one), thus defining the sign of the deviation]. Since the two bars are indistinguishable, the distances are always positive (unlike in figures 3 and 4). Again, the solid lines indicate the best fit through the data points. The corresponding slopes and r^2 values are given in table 2.

The trends that can be seen in figure 6 and table 2 are very similar to those found in the parallelity experiment. The deviations of subjects ME and NK show a high correlation with horizontal distance: the larger the distance the more negative the deviation. Interestingly, for all three subjects the slopes are almost identical to those found in the parallelity experiment. The deviations as a function of vertical distance do not show any correlation.

Figure 5. Results of subject ME in the collinearity experiment. In each of the squares, the small dot indicates the position of the subject. The upper four rows show the average settings of the two test bars; in each square a different combination of test positions is shown. The lower four rows give the deviations in degrees of the test bars averaged over the three settings. Positive and negative numbers indicate counterclockwise and clockwise deviations, respectively. If the subject responded veridically, the two lines in each square would be collinear and all numbers would be zero. Locations of subject and bars, and length of bars are drawn to scale.

Table 2. Slopes and r^2 values for the deviations (in degrees) as a function of distance in the collinearity experiment. The asterisks indicate the significance level of the slopes: *, $p < 0.05$; **, $p < 0.0001$.

Subject	Horizontal distance		Vertical distance	
	Slope/° m^{-1}	r^2	Slope/° m^{-1}	r^2
ME	−34**	0.80	−1	0.00
NK	−20**	0.71	−3	0.00
RR	−2	0.03	−7	0.09

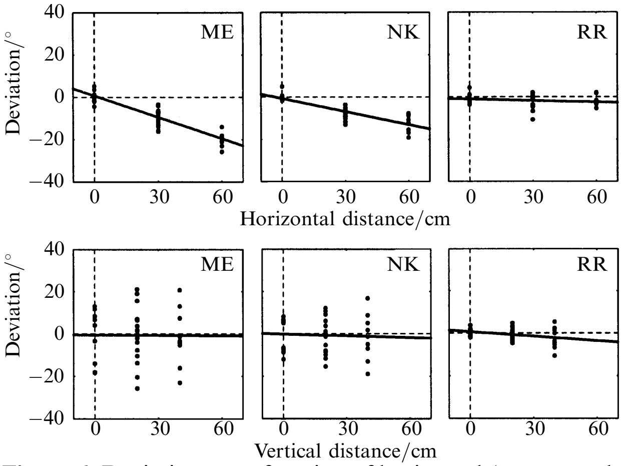

Figure 6. Deviations as a function of horizontal (upper graphs) and vertical (lower graphs) distance between the two bars in the collinearity experiment. Deviations are defined as the orientation of the rightmost bar minus the orientation of the other bar; or, in the case of the vertical distances, as the orientation of the lower bar minus the orientation of the upper bar. The solid line indicates the best fit (in the least-squares sense) through the data points. Slopes, r^2, and significance levels are given in table 2.

4 Experiment 3

In the third experiment—the so-called 'pointing' experiment—the ability of subjects to direct a bar towards a target was investigated. If the collinearity task can be considered as a double pointing task, namely the pointing of two bars towards each other, we could once more expect significant deviations from veridical.

4.1 *Methods*

4.1.1 *Apparatus and subjects.* Again the same setup could be used. One of the bars was replaced by a marker which consisted of a circular magnet with a diameter of 2.3 cm. Under this magnet a small pin was attached which fitted in the holes in the centres of the protractors. In this way, movements of the marker were avoided.

Again, subjects ME, NK, and RR who participated in experiments 1 and 2 participated here as well.

4.1.2 *Stimuli.* All nine possible positions were used as a location for the marker. After a position for the marker was allocated, the remaining eight positions were used for the bar. The total number of combinations is 9 (marker positions) $\times$ 8 (bar positions) $=$ 72. All 72 combinations were presented three times in random order.

4.1.3 *Procedure.* The experimenter placed the right hand of the blindfolded subject first on the marker and next on the bar. The subject was instructed to rotate the bar in such a way that it felt as if it pointed to the centre of the marker in a straight line. The subject could go back and forth from marker to bar as often as needed to perform the task. When the subject was satisfied with the orientation of the bar, the experimenter wrote down the orientation of the bar.

Blocks of trials from this experimental task were mixed with blocks from experiments 1 and 2. The duration of a session was always about 1 h. Subject ME needed 1.5 h to complete this task, NK 4.5 h, and RR 2 h.

4.2 *Results*

In figure 7 the results of subject ME are given. In each square, the little dots indicate the position of the subject [coordinates (0, 0)] and the large dots represent the positions of the marker. The lines in the left-hand squares give the average settings of the test

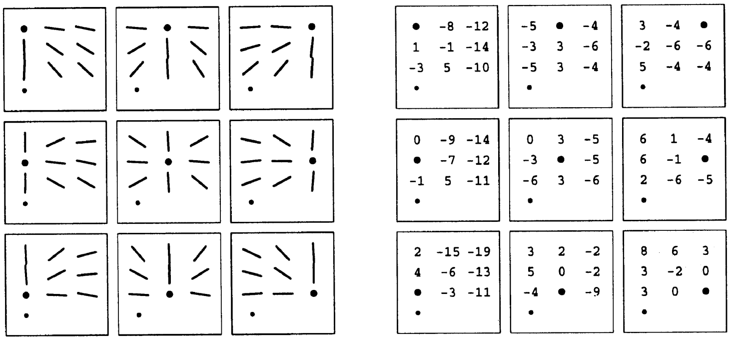

Figure 7. Results of subject ME in the pointing experiment. The nine squares on the left give a graphical representation of the results, whereas the nine squares on the right give a numerical representation of the same results. In each of the squares, the small dot indicates the position of the subject and the large dot the position of the marker. The lines in the left-hand squares indicate the orientations of the test bars averaged over the three settings. The numbers in the right-hand squares give the deviation in degrees of the average settings of the test bars. Positive and negative numbers indicate counterclockwise and clockwise deviations, respectively. Locations of subject, marker, and bars, and length of bars are drawn to scale.

bars and the numbers in the right-hand squares give the corresponding deviations in degrees. Locations of subject, marker, and bars, and length of bars are drawn to scale. The averages of the absolute values of the deviations are 5.1°, 3.2°, and 2.8° for subjects ME, NK, and RR, respectively. The average standard deviations are 3.8°, 2.1°, and 2.3°, respectively. As in the two previous experiments, the standard deviations are smaller than the average deviations, indicating that the subjects did not respond veridically. The underlying patterns of deviations are not as easy to discern as in the parallelity and collinearity experiments, but the trend is similar: pointing from right to left results mostly in clockwise deviations, whereas pointing in the other direction leads to rotations in the counterclockwise direction.

In figure 8 we plotted the deviations as a function of horizontal (upper graphs) and vertical (lower graphs) distances. The solid lines give the best fit (in the least-squares sense) through the data points. In table 3 the corresponding slopes and the r^2 values

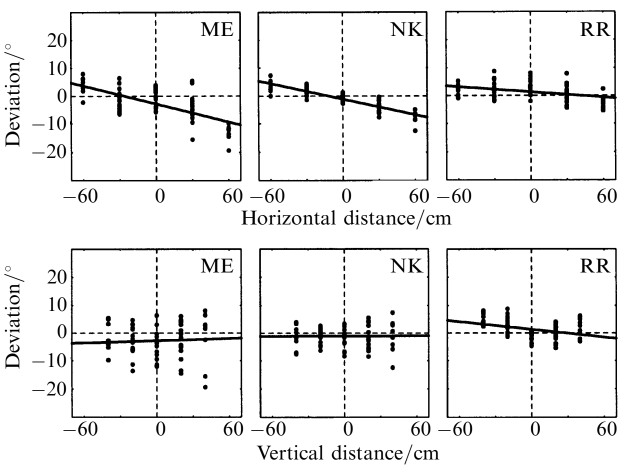

Figure 8. Deviations as a function of horizontal (upper graphs) and vertical (lower graphs) distance between the bar and the marker in the pointing experiment. A negative horizontal distance indicates that the bar is located to the right of the marker, and vice versa. A negative vertical distance indicates that the bar is located further away from the subject in vertical direction than the marker, and vice versa. Slopes, r^2, and significance levels are given in table 3.

Table 3. Slopes and r^2 values for the deviations (in degrees) as a function of distance in the pointing experiment. The asterisks indicate the significance level of the slopes: *, $p < 0.05$; **, $p < 0.0001$.

Subject	Horizontal distance		Vertical distance	
	Slope/° m^{-1}	r^2	Slope/° m^{-1}	r^2
ME	−11**	0.45	1	0.00
NK	−9**	0.79	0	0.00
RR	−3*	0.12	−5*	0.12

are given. Again, for subjects ME and NK the deviations do correlate with horizontal distance but not with vertical distance. The slopes in the horizontal case are negative, just as in the parallelity and collinearity experiments. Subject RR is close to veridical.

5 Discussion

In all three experiments—the parallelity experiment, the collinearity experiment, and the pointing experiment—most subjects' settings were found to deviate significantly from veridical. The patterns of deviations were similar for the three different tasks and the various subjects: there was a high linear correlation between the deviation and the horizontal distance between the two bars or the bar and the marker, whereas there was hardly any correlation between the deviation and the vertical distance. The subjects differed in the size of the deviations which ranged from almost zero (subjects RR and MW) to as much as $-34°$ m^{-1} (the distance here is the horizontal distance) for subject ME. All subjects showed a significant haptic oblique effect in the parallelity experiment. These results falsify the hypothesis that haptic space is Euclidean.

The results of the three different tasks can be compared in a number of ways. First, we can compare the mean absolute errors. In doing so, we should keep in mind that the deviations in the parallelity experiment signify a difference in orientation between two bars, that in the collinearity experiment the deviations of the individual bars are given, and that in the pointing experiment only one bar is present. Thus if the deviations in the various tasks are related, one should expect that the mean absolute errors in the parallelity experiment should be twice as high as the errors in the other two experiments. This expectation is approximately borne out. Second, we can compare the average standard deviations. Once again, one should expect the values in the parallelity experiment to be higher (by a factor $\sqrt{2}$). The tendency in the data certainly points in that direction. Finally, probably the best way to compare the experiments is by looking at the slopes as given in the three tables. In the pointing experiment only one bar is involved; thus, if the tasks are related, the slopes should be half those found in the other two experiments (for the collinearity the deviation is now taken as the difference in orientation between the two bars, just as in the parallelity case). Again, in the horizontal case, the slopes show this pattern (because subject RR is almost veridical, his data cannot contradict this pattern). As expected, in the vertical case the slopes are not significantly different from zero. These results are most parsimoniously explained by the assumption that in the three tasks the deviations are caused by the same underlying mechanism.

It is not at all obvious what causes the horizontal gradient (that is the dependence of the deviation on the horizontal distance or, in this case, the slopes) in the settings of the subjects. Interestingly, if subjects are allowed to inspect their haptic settings visually (of course not during the course of our experiments), they immediately become aware of their deviations, but, if they close their eyes again, the haptically set bars still feel parallel. Thus, an explanation of the deviations in terms of proprioceptive drift (that is, blindfolded subjects lose sight of the world and thus can no longer use visual calibration of their hand and arm positions) is ruled out.

Explanations in terms of rotations of the arm about the shoulder or the elbow, or of the hand about the wrist, cannot account for such a gradient. The expected deviations from veridical would have to be much larger than actually found. If, for example, the hand moves from position (0°, 20°) to position (30°, 60°) the orientation of the hand changes about 90° whereas our subject with the largest deviations (ME) makes an error of "only" about 20°. Neither could simple arm movements explain the large difference in the size of the gradient between the subjects. Although arm lengths and shoulder widths of the subjects are not identical, the differences are only small and subjects RR and MW who were almost veridical fall between the other subjects. The absence of a vertical component in the deviations might find a partial explanation in the fact that forward movements by the subject were not mechanically restricted. For most stimulus pairs such movements were not necessary (and usually not made), but for the most distant locations subjects had to reach forward. Possibly, such movements reduced the influence of a vertical component and this should be tested in some future experiment.

How do our results compare to those of Blumenfeld (1937)? In section 1 we already stated that our experiments are different from his in almost all aspects making comparison a delicate matter. Still, a few observations can be made. The main agreement is that we also find evidence for the deformation of haptic space. Our results can be characterised by a horizontal gradient; a vertical component seems lacking. At first sight, it seems as if Blumenfeld's findings contradicted ours since his vertical alleys are clearly non-parallel but one should keep in mind that his subjects had to construct simultaneously two alleys separated by a horizontal distance. Thus a deviation in horizontal direction is certainly present in his data. However, if we try to extrapolate our results to his bimanual task, we would predict alleys converging towards the median plane which is not in agreement with his results. Indeed, the natures of the two tasks are too different to allow such an extrapolation. It would be of interest to investigate which of the many aspects that are different in the two experiments (his and ours) causes this discrepancy. The consistency we found between the results from the three different experiments argues against a pronounced task-dependent deformation of haptic space, although we should keep in mind that our tasks might be too related to validate such a conclusion.

Although our experiments were not intended to investigate a possible haptic oblique effect, our data from the parallelity experiment allowed an analysis as a function of the reference orientation. The prerequisite for a haptic oblique effect is worse performance with oblique reference orientations (45° and 135°) in comparison with horizontal (0°) and vertical (90°). Performance can be judged both in terms of accuracy and of veridicality. The average standard deviation is a suitable measure for the accuracy. This is, however, not a measure which is usually considered in studies of a haptic oblique effect. Veridicality is usually assessed by either the mean signed error (eg Lechelt et al 1976; Gentaz and Hatwell 1995) or the mean absolute error (eg Lechelt et al 1976; Lechelt and Verenka 1980; Appelle and Gravetter 1985; Appelle and Countryman 1986; Gentaz and Hatwell 1995). In our experiments it did not make sense to look at the mean signed error, since the sign depends on the relative positions of the test and reference bars. For all subjects we found that the mean absolute error is significantly larger for the oblique reference orientations than for horizontal or vertical orientations. Moreover, the slopes shown in figure 3 and table 1 are often clearly dependent on reference orientation.

A factor which should be taken into account in this respect is that in cases where the reference orientation was 90° and the two bars were separated 20 cm in vertical direction (with the same horizontal location), the two bars almost touched. Although the subjects were not allowed to touch the two bars at the same time, this could not always be avoided; unintentionally, this could have influenced the size of the deviations in the case of the 90° reference orientation. However, this argument does not count for subjects RA and MK since in their experiment a vertical distance of 20 cm did not occur and it is they who show the most prominent oblique effect. Thus, we conclude that our data clearly give evidence for the existence of a haptic oblique effect.

For the time being we are satisfied with the conclusion that haptic space is not Euclidean. Our data suggest that a description in terms of a horizontal gradient in the deviations might be a useful first step in formalising the deformation of haptic space, but clearly more research is needed. The validity of such a description should be investigated over a larger part of haptic space (eg left of the median plane, frontal plane, etc), be tested for both unimanual and bimanual performance, and be measured under more restricted conditions (such as the restriction of forward movements). Eventually, we hope to understand more of the underlying mechanisms causing the systematic deformation of haptic space.

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